

MATH 110 Automotive Worksheet #4

Ratios

The math name for a fraction is *ratio*. It is just a comparison of one quantity with another quantity that is similar. As an automotive technician, you will use ratios a lot.

You can use ratios when you convert from one unit to another. Since 12 inches = 1 foot, the conversion ratio can be written as a fraction

$$\frac{12 \text{ inches}}{1 \text{ foot}} \quad \text{or} \quad \frac{1 \text{ foot}}{12 \text{ inches}}$$

You can see that these ratios relate inches and feet.

Example 1: Using a ratio to convert from 2.5 feet into an equivalent number of inches, we use the first ratio.

$$(2.5 \text{ feet}) \times \left(\frac{12 \text{ inches}}{1 \text{ foot}} \right) = 30 \text{ inches}$$

So 2.5 feet equals 30 inches. Notice the feet units cancel (because $\frac{\text{feet}}{\text{feet}} = 1$; that is, anything divided by itself is equal to 1). Always arrange the number you want to convert and the unit conversion ratio so that the units you don't want cancel out and you are left with the units you do want in the numerator.

Example 2: Using a ratio to convert from 42 inches into an equivalent number of feet, we use the second ratio.

$$(42 \text{ inches}) \times \left(\frac{1 \text{ foot}}{12 \text{ inches}} \right) = 3.5 \text{ feet}$$

So 42 inches equals 3.5 feet.

Notice that in each case, you use the conversion ratio that forces the units to cancel (divide) out. The units tell you which form of the conversion ratio you must use.

Ratios in the Automotive Industry

Ratios are often written with a colon “:”

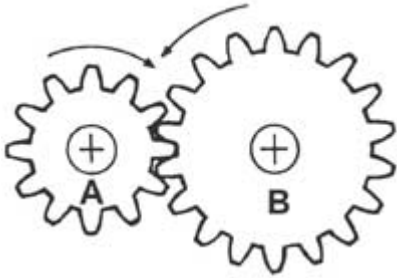
For instance, to express the windings of a coil with 1600 turns and 400 turns as a ratio you would say 1600:400, which can then be reduced to 4:1 since you can divide 400 into both sides of the ratio.

A *gear ratio* compares the number of teeth on different gears. It is the comparison of the number of teeth on the *input* gear to the number of teeth on the *output* gear (the input gear is the one the power is applied to, also called the *driver* gear. The output gear is also called the *driven* gear). To find the ratio, you should divide the number of teeth of the input gear by the number of teeth of the output gear. This is usually

stated as (the answer for the division):1. If the gear ratio of **A** to **B** is 3:1, then gear A has three times as many teeth as gear B.

$$\text{gear ratio} = \frac{\text{number of teeth in input gear}}{\text{number of teeth in output gear}} =$$

$$= \text{number of teeth in input gear} : \text{number of teeth in output gear}$$

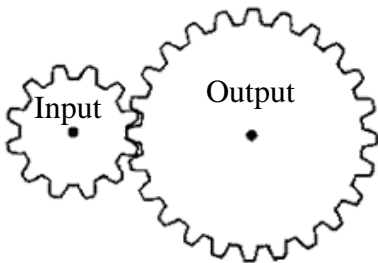


In this figure the gear ratio A to B is 12:18 or, after you reduce it by dividing both sides by 12 (the smaller of the two numbers), you get a ratio of 1:1.5

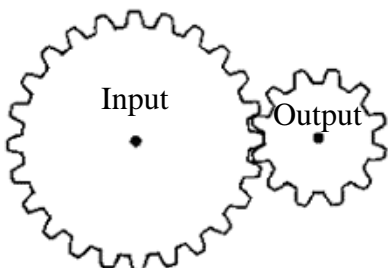
The principle behind gears is pretty simple. **Gears with unequal numbers of teeth alter the speed between the input and out put gears.** Say the gear ratio of the input gear to the output gear is 1:3 (like 10 teeth for gear A and 30 teeth for gear B). Then for every complete revolution of the input gear the output gear turns 1/3 the way around. This means you are slowing down the action (this is referred to in engineering terms as “Stepping Down”.) If we reverse everything so that the A has 30 teeth and B has 10, then the opposite happens and we “Step Up”-- then for every 1 turn of the input gear the output gear would turn 3 times and the ratio is now 3:1. This means the formula for the gear ratio can also be written as

$$\text{gear ratio} = \frac{\text{number of revolutions of the output gear}}{\text{number of revolutions of the input gear}} =$$

$$= \text{number of revolutions of the output gear} : \text{number of revolutions of the input gear}$$



Here the input gear turns faster than the output gear. Stepping down has the advantage of producing more power although at a slower rate.



Here the output gear turns faster than the input gear. Stepping up produces a much faster output speed, but mechanically delivers less power.

Example 3: The input gear has 75 teeth and the output gear has 10 teeth. What is the gear ratio?

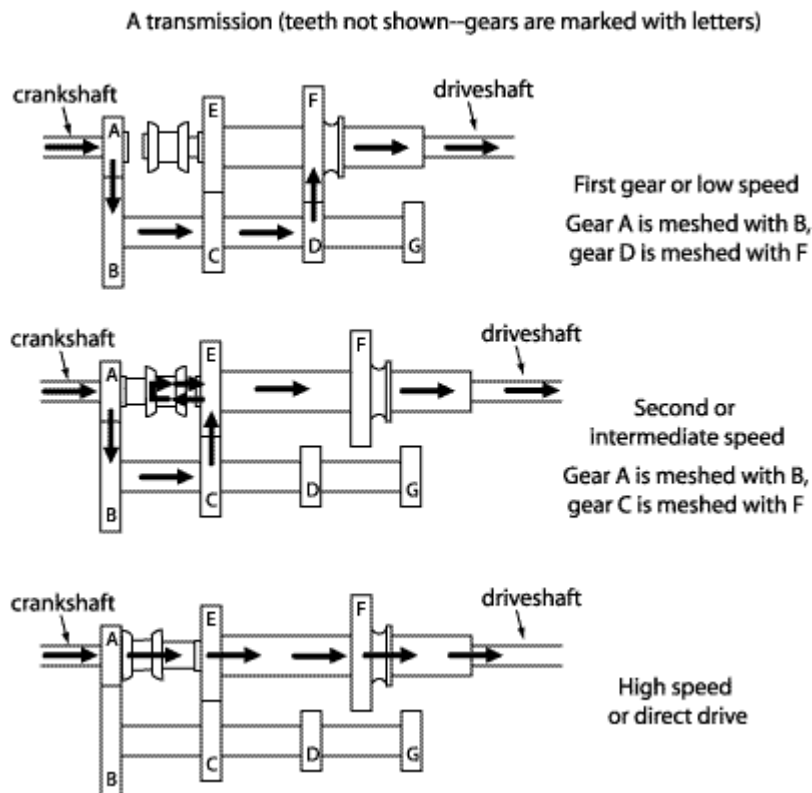
Solution: From the first gear ratio formula listed above, the ratio of the number of teeth in the input to the number of teeth in the output is $\frac{75}{10} = 7.5$. So the gear ratio is 7.5:1.

Example 4: The gear ratio of the teeth on input gear A to output gear B is 6:1. How many times faster does gear B move than gear A?

Solution: From the second gear ratio formula listed above, the ratio of the number of revolutions of A to B is 1:6. That is, for every complete turn of A, B turns 6 times. Therefore, B turns 6 times faster than A.

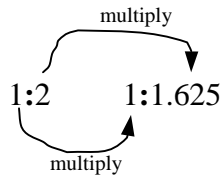
The *transmission gear ratio* refers to the number of times the speed is reduced by the transmission. It compares the speed of the crankshaft to the speed of the driveshaft. A transmission ratio of 3:1 means the speed is reduced three times by the transmission (the RPMs of the crankshaft is three times the RPMs of the driveshaft).

To find the transmission gear ratio, first find the gear ratio of each set of gears that are meshed together. Then multiply the first numbers of the gear ratios together and compare to the product of the second numbers. Then reverse the numbers. In the figure below, the arrows show the direction the power is applied.



Example 5: In low speed, gears A and B, and gears D and F are meshed. A has 14 teeth, B has 28 teeth, D has 16 teeth and F has 26 teeth. What is the transmission gear ratio?

Solution: First find the gear ratio for each set of meshed gears. For A and B, the gear ratio is 14:28 or 1:2. For gears D and F, the gear ratio is 16:26 or after dividing both sides by 16 gives, 1:1.625. To get the transmission gear ratio multiply the first numbers in each ratio together and do the same for the second set of numbers then compare. That is

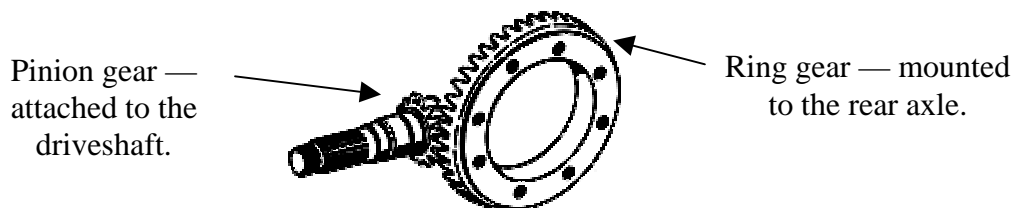


or $1 \times 1:2 \times 1.625$ which gives us 1:3.25. You then reverse the numbers to get 3.25:1. This means the transmission reduces the speed by a factor of 3.25 (the driveshaft spins 3.25 times slower than the crankshaft).

Example 6: In second gear, gears A and B and gears C and E are meshed. A has 14 teeth, B has 28 teeth, C has 21 teeth and E has 21 teeth. What is the transmission gear ratio?

Solution: The gear ratio of each set of gears is: For A and B: 1:2. For C and E: 1:1. Then we multiply the numbers from each ratio: $1 \times 1:2 \times 1$ which gives us, 1:2. Then reverse the numbers to get the transmission gear ratio: 2:1. This means the crankshaft spins 2 times faster than the driveshaft.

The *axle or differential ratio* refers to the number of times the speed of the driveshaft is reduced by the ring gear and pinion. It compares the speed of the driveshaft to the speed of the rear axle shaft. An axle ratio of 4:1 means the RPM's of the driveshaft is 4 times greater than the RPM's of the rear axle shaft. To find the axle ratio, find the ratio of the number of teeth of the ring gear to that of the pinion gear. See the figure:



Example 7: The pinion gear has 12 teeth and the ring gear has 42 teeth. What is the axle ratio?

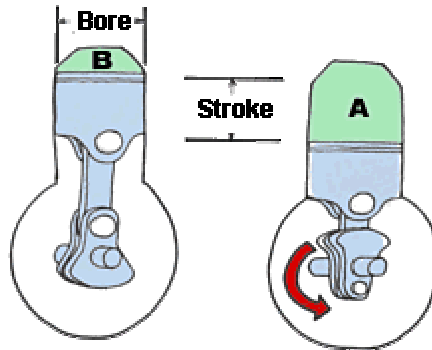
Solution: The axle ratio is the ratio of the number of teeth of the ring gear to that of the pinion gear. So we have 42:12. Divide both sides by 12 to reduce it. That gives us $42/12:1$ or 3.5:1. This means the driveshaft turns 3.5 times faster than the rear axle shaft does.

The *total gear reduction* is the ratio of the RPM of the crankshaft to the RPM of the rear axle shaft. In finding the total gear reduction, both the transmission gear ratio and the axle (differential) ratios are used. Multiply the first numbers from both ratios and then multiply the second set of numbers. Then compare them.

Example 8: A transmission gear ratio is 2:1 and the axle ratio is 3:1. What is the total gear reduction ratio?

Solution: The total gear reduction ratio is $3 \times 2:1 \times 1$, or $6:1$.

Another type of ratio you will run across in your work is the *compression ratio*. The compression ratio is a comparison to the amount of space (the volume in cubic inches or cubic centimeters-- cc) in the cylinder when the piston is at the bottom of the stroke (indicated by A in the drawing), and the amount of space when the piston is at the top of the stroke (indicated by B in the drawing). For example, if there is 8 times as much space when the piston is at the bottom of the stroke as when the piston is at the top of the stroke, the compression ratio is $8:1$.



Example 9: There are 33 cubic inches of space when the piston is at the bottom of the stroke and 4 cubic inches of space when the piston is at the top of the stroke. What is the compression ratio?

Solution: The compression ratio is $33:4$ or after dividing both sides by 4, $8.25:1$.

Proportions

A proportion is two ratios set equal to one another. A proportion is a fraction equal to a fraction. For example, $3:1 = 12:4$ is the same thing as

$$\frac{3}{1} = \frac{12}{4}.$$

The above problems can be done using proportions.

Example 10: An input gear turns three revolutions while the output gear only turns one revolution. The larger gear has 27 teeth. How many teeth does the smaller gear have?

Solution: The gear ratio is $1:3$ since the input gear turns three times as fast as the output gear. The larger gear with 27 teeth has to be the output gear since it turns more slowly than the input gear. We can set the gear ratio equal to the ratio of the input gear's teeth to that of the output gear's teeth: $1:3 = X:27$, where X represents the unknown number of teeth in the input gear. This proportion can be written as two equal fractions:

$\frac{1}{3} = \frac{X}{27}$. You can solve this for X by cross multiplying. To do this, multiply the denominators of each fraction by the numerators of the other and set them equal to each other leaving: $(1)(27) = 3(X)$. This can be solved for X by dividing both sides by 3:

$$\frac{(1)(27)}{3} = \frac{(\cancel{3})(X)}{\cancel{3}}$$

which leaves us
9 = X

Here you could have saved a step by just multiplying the 1 by the 27. That would leave you

$$\frac{(1)(27)}{3} = X \text{ which reduces to } X = 9. \text{ So the smaller input gear has 9 teeth.}$$

Homework Problems

1. Convert 57 inches to feet. State the answer two ways: just feet (in decimal form, like 2.3 feet), and feet and inches (like 3 feet 4 inches).
2. 1 inch = 25.4 millimeters. How many millimeters are there in 2.7 inches?
3. An input gear has 55 teeth and the output gear has 25 teeth. What is the gear ratio of these gears? Which gear turns faster and how much faster is it compared to the other gear?
4. The gear ratio of two gears A and B is 6.2:1. How many times slower does gear A move compared to gear B?

Refer to the transmission diagram earlier in this document for the next two problems.

5. What is the transmission gear ratio when the car is in second gear (intermediate speed), assuming gear A has 12 teeth, gear B has 36 teeth, gear C has 30 teeth and gear E has 24 teeth?
6. In high speed gear there are internal splines that mesh with the teeth on the back of the gear A. This gives direct drive. What is the transmission gear ratio?
7. A ring gear has 42 teeth and the pinion gear has 16 teeth. What is the axle ratio, and how much slower does the rear axle shaft spin compared to the driveshaft?
8. The second gear transmission gear ratio of a truck is 1.9:1. The rear axle ratio is 4.6:1. Find the total gear reduction. Also, how much faster does the crankshaft turn than the rear axle shaft?
9. A sport truck with a 2.3 Liter turbocharged diesel engine has a cylinder volume at bottom dead center of 605 cc. At top dead center, the volume is 30 cc. What is the compression ratio?
10. A gear with 15 teeth turns as 160 RPM. It is driving a gear of 25 teeth. Find the RPM of the driven gear.